Name:____________________________

MA 3053 Section 01 *Practice Exam 1* *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let $f : X \to X$. Suppose f has the property that $f \circ f = \mathrm{id}|_X$, that is $(f \circ f)(x) = x$ for all $x \in X$. Prove that f is a bijection.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a decreasing function. Prove that f is an injection.

3. Let $f : X \to Y$ and $P_\alpha \subseteq Y$ for every $\alpha \in A$ Show

$$f^{-1}\left(\bigcup_{\alpha \in A} P_\alpha\right) = \bigcup_{\alpha \in A} f^{-1}(P_\alpha)$$

4. Let $f : X \to Y$ be an injection and $P_\alpha \subseteq X$ for every $\alpha \in A$. Show that

$$f\left(\bigcap_{\alpha \in A} P_\alpha\right) = \bigcap_{\alpha \in A} f(P_\alpha)$$

5. Let $\preceq$ be the relation on $\mathbb{N}^+$ defined by $x \preceq y$ if and only if there is a $z \in \mathbb{N}^+$ such that

$$xz = y\,.$$

Prove that $\preceq$ is a partial ordering on $\mathbb{N}^+$.

6. Let X be a set and $P = \{f : f : X \to X\}$. Define the relation $\preceq$ on P by $f \preceq g$ if and only if $f(x) \leq g(x)$ for all $x \in X$. Prove $\preceq$ is a partial order on P.

7. Let $f : X \to X$ be a function. Define the relation $\sim$ on X by $x \sim y$ if and only if $f(x) = f(y)$. Prove that $\sim$ is an equivalence relation on X. What are the equivalence classes in the quotient space $X/\sim$, be sure to justify.

8. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{N}^+$ by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show $\sim$ is an equivalence relation on X.

9. What are the multiplication and addition tables for the congruence classes in $\mathbb{Z}/6\mathbb{Z}$.